On Lindley-Pareto Distribution: Properties and Application

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***Abstract*. A new distribution is proposed for modeling life-time data, called as Lindley Pareto distribution (LP). Various statistical properties like the quantile function, moment method, maximum likelihood estimation.**

**Keywords; Estimation, moments, T-X family, Lindley distribution, Pareto distribution.**

1. INTRODUCTION

The real-life applications of contemporary numerical techniques in different fields such as medicine, finance, biological engineering sciences and statistics. To this end, statistics plays a crucial role in real life applications. Often by using the statistical analysis which strongly depends on the assumed probability model or distributions. However, several problems in statistics does not follow any of the classical or standard probability.

Let be a random variable following the one parameter Distribution with the density function



introduced by Lindley [11]. Sankaran [17] used (1) as mixing distribution of Poisson parameter which it named Poisson- Lindley distribution. Recently, As-gharzadeh et al. [4], Ghitany et al. [6] and [7] re-discovered and studied the new distribution bounded to (1), what they derived is known as Zero-truncated Poisson- Lindley and Pareto Poisson-Lindley distributions.

 Furthermore, Pareto distribution was pioneered by V. Pareto [13] to explore unequal distribution of wealth. It is widely used in actuarial science.(e.g. reinsurance) because of its heavy tail properties. To add flexibility to the Pareto distribution, various generalizations of the distribution have been derived, including: the generalized Pareto distribution (Pickands [14]), the beta-Pareto distribution (Akin-sete et al.[1]), and the beta generalized Pareto distri-bution (Mahmoudi [12]).

 The mixed distribution is one of the most important ideas for obtaining a new distribution. For example, Sharma and Shanker [18] used a mixture of exponential (θ) and gamma (2, θ) to create a two-parameter Lindley distribution. For another example, Zakerzadeh and Dolati [19] used gamma (α, θ) and gamma (α + 1, θ) to create a generalized Lindley distribution. Recently, Zeghdoudi and Nedjar [20,21] introduced a new distribution, named gamma Lindley distribution, based on mixtures of gamma (2, θ) and one-parameter Lindley distributions. In addition, the CDF of the T-X family of distributions defined by Alzaatreh, et al. [2] is given by



is another idea for obtaining a new distribution. In this paper, we introduce a new family of distribution generated by a random variable T which follows the Lindley distribution with one parameter θ > 0, then



and the definition in (1) leads to the Lindley-Xfamily with PDF



We consider F (x) corresponding to Pareto distribution with CDF



Hence the cumulative distribution function of the new distribution is given by



with corresponding density



 We refer the random variable with cumulative distribution function (4) as Lindley Pareto (LP) distribution with parameters θ, α and k which we denote it by LP(θ,α,k ).

Of in Sec. 2, we study the properties like shapes of the pdf. Moments and the moment-generating function are studied in Sec. 3 and in Sec. 4 quantile function. In sec. 5 extreme order statistics, entropy in sec 6. Section 7 the maximum likelihood estimation.

1. Properties of the Lindley Pareto distribution
2. *Shape of the density*

 The derivative with respect to x of Eq. (3) is given by :



For the probability density function of the LP distribution is decreasing.

For and ,  is the unique critical point which LP distribution is maximum.

 The mode x0 is the solution of equation, where



Therefore, the mode of L distribution is given by

mode for  ,  , and mode otherwise.

The hazard function associated with LP distribution is



The limiting behaviors of the pdf and hrf of X are given in Theorem 1.

**Theorem 1.** The limit of the pdf as x → ∞ is 0. Also, the limits of the pdf and hrfof X as x → α+ are given by



 Further, the limits of the hrf of X as x → ∞ are given by



1. Maximum Likelihood Estimates (MLE)

Let , be n random variables. The ln-likelihood function  is:



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1. Application to real data sets

We give, the applicability of LP distribution by considering the data sets used by different researchers: Application to waiting times in a queue and compared with different distribution of which Lindley exponential, Lindley Weibull, Lindley, GaL, Power Lindley [6], exponential Pareto, Pareto and gamma Lindley distributions. In each case, the parameters are estimated by maximum likelihood, using the R software.

In order to compare the above distributions with Lindley Pareto distribution, we consider criteria like for the data set. The model selection is carried out using the following statistics:



 

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1. Illustration 1: Application to waiting times in a queue

 We consider 100 observations on waiting time as a second example that happens before the customer received service in a bank. The data sets are represented in Appendix 2 as Table 3 and its result are represented in table 2.

Table 1 Parameter estimates for 100 bank customers

|  |  |
| --- | --- |
| Distribution | Parameters |
| LP |  |
| LE |  |
| EP |  |
| GaL |  |
| L |  |
| P |  |
| LW |  |
| PL |  |

Table 2 The -LL, AIC, CAIC, BIC for 100 bank customers

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Distributions | **-LL** | **AIC** | **CAIC** | **BIC** |
| **LP** | 308.97 | 621.94 | 622.08 | 627.63 |
| LE | 317.00 | 638.01 | 638.13 | 643.22 |
| **EP** | 312.11 | 628.23 | 628.37 | 633.91 |
| **GL** | 317.30 | 638.61 | 638.73 | 643.82 |
| **L** | 319.00 | 640.00 | 640.04 | 642.60 |
| **P** | 381.75 | 765.51 | 765.56 | 767.99 |
| **LW** | 317.32 | 640.65 | 640.90 | 648.46 |
| **PL** | 318.31 | 640.63 | 641.91 | 645.84 |

1. Conclusion

 We have proposed the new distribution Lindley- Pareto(L-P) distribution generated by Lindley distribution. Various statistical properties like the quantile function, moment method, maximum likelihood estimation, entropy, and limiting distribution of extreme order statistics are established. A simulation study is carried out to examine the quantiles, mean, median and mode of the Lindley Pareto distribution.

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